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Mirage Cosmology in M-theory

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Abstract

We extend the idea of mirage cosmology to M-theory. Considering the motion of a probe brane in the M-theory background generated by a stack of non-threshold (M2,M5) bound states, we study the cosmological evolution of the brane universe in this background. We estimate the range of r where the formalism is valid. Effective energy density on the probe brane is obtained in terms of the scale factor. Comparing the limiting case of the result with that from type IIB background, we confirm that the cosmological evolution by mirage matter is a possible scenario in the M-theory context.

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I. INTRODUCTION

Recently there has been renewed interest in the cosmological model based on the brane universe since this idea can be applied to string theory. The idea of brane universe is that our observed universe is a three-brane embedded in a higher dimensional space [1, 2]. Many cosmological models regarding this have been studied. These models can largely be classified into two categories. One is that the brane is a static solution of the underlying theory and the cosmological evolution is due to the time evolution of the energy density on the brane [3]. The other is that the cosmological evolution of the brane universe is due to the motion of the brane in the background of the bulk as well as the matter density on the brane [4, 5, 6, 7].

One interesting model among the second category is the so-called mirage cosmology presented by Kehagias and Kiritsis [5]. The idea is that the motion of the brane through the bulk, ignoring its back reaction to the ambient geometry, induces cosmological evolution on the brane even when there is no matter field on the brane. The crucial mechanism underlying the construction of this formalism is the coupling of the probe brane to the background gauge field. They derived Friedman-like equations for various bulk background field solutions within type II string theory.

This model was studied extensively by others. The mirage cosmology with non-trivial dilaton field was studied by the author [8]. Since the dilaton as well as the induced metric affects the effective matter density, the cosmological evolution with nontrivial dilaton profile is different from the one without dilaton. The motion of a three-brane in the background of type 0B string theory was examined in Ref. [9]. Brane inflation for tachyonic and non-tachyonic type 0B string theories was studied and it is known that the presence of tachyon slows down the inflation in mirage cosmology. Brane cosmology in the background of D-brane with NS B field was studied by Youm [10]. The corrections to the Friedman equations due to nonzero NS B field were obtained and analyzed for various limits. The mirage cosmology for non-planar probe universe was studied in Ref. [11]. There the author considered the spherical probe brane wrapped around the sphere part of various background spacetimes and commented its relevance to the giant graviton [12]. It is known that the mirage cosmology approach matches with the familiar junction condition approach when there is just one extra dimension [13].

Since type 0B string theory is defined on the world sheet of type IIB theory by performing a nonchiral Gliozzi-Sherk-Olive (GSO) projection [14], so far the study on mirage cosmology is mainly based on the type IIB string theory. In this paper we will extend the idea of mirage cosmology to M-theory. As a concrete example, we consider the M-theory background generated by a stack of non-threshold (M2,M5) bound states. We study the cosmological evolution by mirage matter in this background.

The organization of the paper is as follows. In Sec. II we briefly review the (M2,M5) background. In Sec. III we construct the action of a probe M5-brane under the background ignoring the back reaction. In Sec. IV, we consider the cosmological evolution of the brane. We estimate the range of r where the formalism is valid. Effective matter density on the probe brane is expressed as a function of the scale factor. We also discuss a limiting case of the result to compare with the known result from the type IIB string background. Finally we conclude and discuss our results in Sec. V.

II. THE BRANE BACKGROUND

The supergravity background we will consider is the one generated by a stack of parallel non-threshold (M2,M5) bound states [15]. The metric for this eleven dimensional supergravity solution can be written as [16]

$$\begin{aligned} ds^2 &= f^{-1/3}h^{-1/3}\Big[-(dx^0)^2+(dx^1)^2+(dx^2)^2+h\Big\{(dx^3)^2+(dx^4)^2+(dx^5)^2\Big\}\Big] \\ &+ f^{2/3}h^{-1/3}\Big[dr^2+r^2d\Omega_4^2\Big], \end{aligned} \tag{1}$$

where $d\Omega_4^2$ is the metric of a unit 4-sphere and f and h are given by

$$f = 1+\frac{R^3}{r^3}, \qquad h^{-1} = \sin^2\varphi f^{-1} + \cos^2\varphi. \tag{2}$$

The above solution appeared in Ref. [17] and was interpreted as a two-brane lying within a five-brane. The M5-brane component extends along the directions $x^0, \cdots, x^5$, while the M2-brane lies along x^0, x^1, x^2. The angle φ in Eq. (2) carries the mixing of the M2- and M5-branes in the bound state. The radial parameter R is defined as $R^3\cos\varphi \equiv \pi N l_p^3$, where l_p is the eleven dimensional Planck length and N is the number of the bound states of the stack. We also have a non-vanishing value of the four-form field strength $F^{(4)}$ given by

$$F^{(4)} = \sin\varphi\partial_r(f^{-1})dx^0 \wedge dx^1 \wedge dx^2 \wedge dr - 3R^3\cos\varphi\epsilon_{(4)}$$
$$- \tan\varphi\partial_r(hf^{-1})dx^3 \wedge dx^4 \wedge dx^5 \wedge dr, \quad (3)$$

where $\epsilon_{(4)}$ represents the volume form of the unit four-sphere S^4. We parameterize the metric of S^4 as

$$d\Omega_4^2 = \frac{1}{1-\rho^2}d\rho^2 + (1-\rho^2)d\phi^2 + \rho^2 d\Omega_2^2, \quad (4)$$

where $d\Omega_2^2$ is the metric of a unit two-sphere S^2 (which we will label θ^1 and θ^2). The ranges of ρ and ϕ are $0 \le \rho \le 1$ and $0 \le \phi \le 2\pi$ respectively. Then, the three-form and six-form potential relevant for our calculation can be written as

$$C^{(3)} = -\sin\varphi f^{-1}dx^0 \wedge dx^1 \wedge dx^2 - R^3\cos\varphi\rho^3 d\phi \wedge \epsilon_{(2)} + \tan\varphi h f^{-1}dx^3 \wedge dx^4 \wedge dx^5, \quad (5)$$
$$\begin{aligned} C^{(6)} &= \frac{1}{2}\sin\varphi\cos\varphi f^{-1}R^3\rho^3 dx^0 \wedge dx^1 \wedge dx^2 \wedge d\phi \wedge \epsilon_{(2)} \\ &- \frac{1}{2}\frac{1-h\cos^2\varphi}{\cos\varphi}f^{-1}dx^0 \wedge dx^1 \wedge dx^2 \wedge dx^3 \wedge dx^4 \wedge dx^5 \\ &- \frac{1}{2}\sin\varphi R^3\rho^3 h f^{-1}dx^3 \wedge dx^4 \wedge dx^5 \wedge d\phi \wedge \epsilon_{(2)}, \end{aligned} \quad (6)$$

where $\epsilon_{(2)}$ is the volume form of S^2. For later use we can write the metric components of Eq. (1) as

$$\begin{aligned} -g_{00} &= g_{11} = g_{22} = f^{-1/3}h^{-1/3}, \qquad g_{33} = g_{44} = g_{55} = f^{-1/3}h^{2/3} \equiv g(r), \\ g_{rr} &= f^{2/3}h^{-1/3}, \qquad g_{\rho\rho} = f^{2/3}h^{-1/3}\frac{r^2}{1-\rho^2}, \qquad g_{\phi\phi} = f^{2/3}h^{-1/3}r^2(1-\rho^2), \\ g_{\theta^1\theta^1} &= f^{2/3}h^{-1/3}r^2\rho^2, \qquad g_{\theta^2\theta^2} = f^{2/3}h^{-1/3}r^2\rho^2\sin^2\theta^1. \end{aligned} \quad (7)$$

III. THE PROBE M5-BRANE ACTION

We consider a probe M5-brane moving in the background of (M2, M5) bound states which shares (x^3, x^4, x^5) directions with the background and wraps S^2 (θ^1, θ^2). The dynamics of M5-brane, ignoring all fermions, is given by the so-called PST action [18]. In PST formalism the world volume fields are a three-form field strength F and a scalar a (the PST scalar). The action consists of three terms

$$S = T_{M5} \int d^6\xi [\mathcal{L}_{DBI} + \mathcal{L}_{H\tilde{H}} + \mathcal{L}_{WZ}], \tag{8}$$

where T_{M5} is the tension of the M5-brane $T_{M5} = 1/(2\pi)^5 l_p^6$. The explicit forms of $\mathcal{L}_{DBI}$, $\mathcal{L}_{H\tilde{H}}$ and $\mathcal{L}_{WZ}$ are given by

$$\begin{aligned} \mathcal{L}_{DBI} &= -\sqrt{-\det(\gamma_{ij} + \tilde{H}_{ij})}, & (9)\\ \mathcal{L}_{H\tilde{H}} &= \frac{1}{24(\partial a)^2} \epsilon^{ijklmn} H_{lmn} H_{jkp} \gamma^{pq} \partial_i a \partial_q a, & (10)\\ \mathcal{L}_{WZ} &= \frac{1}{6!} \epsilon^{ijklmn} \Big\{ P[C^{(6)}]_{ijklmn} + 10 H_{ijk} P[C^{(3)}]_{lmn} \Big\}, & (11) \end{aligned}$$

where γ is the induced metric on the M5-brane worldvolume

$$\gamma_{ij}(\xi) = g_{\mu\nu}(x) \frac{\partial x^\mu}{\partial \xi^i} \frac{\partial x^\nu}{\partial \xi^j}, \tag{12}$$

and $P[C^{(3)}]$ and $P[C^{(6)}]$ are the pullbacks of the corresponding background potentials. The field H and $\tilde{H}$ are defined as

$$\begin{aligned} H_{ijk} &= F_{ijk} - P[C^{(3)}]_{ijk}, & (13)\\ \tilde{H}^{ij} &= \frac{1}{3!\sqrt{-\det\gamma}} \frac{1}{\sqrt{-(\partial a)^2}} \epsilon^{ijklmn} \partial_k a H_{lmn}. & (14) \end{aligned}$$

To write down the action explicitly, we take the worldvolume coordinates $\xi^i (i = 0, 1, \cdots, 5)$ in the static gauge as

$$\xi^i = (x^0, x^3, x^4, x^5, \theta^1, \theta^2). \tag{15}$$

In this system of coordinates the variables x^1, x^2, r, ρ, ϕ are functions of ξ^i in general. We assume that these variables depend only on time and there is a translational symmetry along the x^1 and x^2 directions. Then the configuration we are interested in is described by

$$r = r(t), \quad \rho = \rho(t), \quad \phi = \phi(t), \tag{16}$$

where $t = x^0$. The induced metric γ_{ij} is calculated, in terms of eleven dimensional spacetime metric, as

$$\begin{aligned}
\gamma_{00} &= -|g_{00}| + g_{rr}\dot{r}^2 + g_{\rho\rho}\dot{\rho}^2 + g_{\phi\phi}\dot{\phi}^2, \\
\gamma_{33} &= \gamma_{44} = \gamma_{55} = g(r), \\
\gamma_{\theta^1\theta^1} &= g_{\theta^1\theta^1}, \qquad \gamma_{\theta^2\theta^2} = g_{\theta^2\theta^2},
\end{aligned}$$

where the dot$(\cdot)$ denotes the derivative with respect to t. We also assume that the only non-vanishing components of H are those of $P[C^{(3)}]$, i.e. $H_{x^3x^4x^5} \equiv H_{345}$ and $H_{x^0\theta^1\theta^2} \equiv H_{0*}$. By fixing the gauge, the auxiliary field a can be eliminated from the action at the expense of losing the manifest covariance. Choosing the gauge $a = x^0 = t$, the only nonzero component of $\tilde{H}$ is

$$\tilde{H}_{\theta_1\theta_2} = \sqrt{\frac{g_{\theta^1\theta^1}g_{\theta^2\theta^2}}{g^3}}H_{345} = f^{7/6}h^{-4/3}r^2\rho^2\sqrt{\hat{g}^{(2)}}H_{345}, \tag{17}$$

with $\hat{g}^{(2)}$ being the determinant of the metric of the unit two-sphere. Using (17) one can calculate $\mathcal{L}_{DBI}$ as

$$\begin{aligned}
\mathcal{L}_{DBI} &= -\sqrt{(|g_{00}| - g_{rr}\dot{r}^2 - g_{\rho\rho}\dot{\rho}^2 - g_{\phi\phi}\dot{\phi}^2)g_{\theta^1\theta^1}g_{\theta^2\theta^2}(g^3 + H_{345}^2)} \\
&= -fr^3\rho^2\sqrt{\hat{g}^{(2)}}\lambda_1\left[r^{-2}f^{-1} - r^{-2}\dot{r}^2 - \frac{\dot{\rho}^2}{1-\rho^2} - (1-\rho^2)\dot{\phi}^2\right]^{1/2},
\end{aligned} \tag{18}$$

where λ_1 is defined as

$$\lambda_1 \equiv \sqrt{hf^{-1} + H_{345}^2h^{-1}}. \tag{19}$$

The remaining terms of the action are calculated as

$$\mathcal{L}_{H\tilde{H}} + \mathcal{L}_{WZ} = \frac{1}{2}F_{345}F_{0*} - F_{345}P[C^{(3)}]_{0*} + P[C^{(6)}]_{0345*} + \frac{1}{2}P[C^{(3)}]_{345}P[C^{(3)}]_{0*}, \tag{20}$$

where the index $0*$ means $x^0\theta^1\theta^2$. The pullbacks of $C^{(3)}$ and $C^{(6)}$ are

$$P[C^{(3)}]_{0*} = -R^3\rho^3\cos\varphi\sqrt{\hat{g}^{(2)}}\dot{\phi},$$

$$P[C^{(3)}]_{345} = \tan\varphi h f^{-1}, \tag{21}$$
$$P[C^{(6)}]_{0345*} = \frac{1}{2}R^3\rho^3 \sin\varphi h f^{-1}\sqrt{\hat{g}^{(2)}}\dot{\phi}.$$

Substituting Eq. (21) in Eq. (20) the last two terms cancel each other, and we have

$$\mathcal{L}_{H\tilde{H}} + \mathcal{L}_{WZ} = R^3\rho^3 F_{345}\cos\varphi\sqrt{\hat{g}^{(2)}}\dot{\phi} + \frac{1}{2}F_{345}F_{0*}. \tag{22}$$

We assume that $F_{0*} = \sqrt{\hat{g}^{(2)}}f_{0*}$ with f_{0*} being independent of the angle of the S^2. With this ansatz for the electric component of F, we can integrate out θ_1 and θ_2 using

$$\int \sqrt{\hat{g}^{(2)}}d\theta_1 d\theta_2 = 4\pi \equiv \Omega_2. \tag{23}$$

Then the action can be reduced to the following four-dimensional (three-brane) effective action

$$S = \int dt dx^3 dx^4 dx^5 \mathcal{L}, \tag{24}$$

with

$$\mathcal{L} = \Omega_2 T_{M5}\Big\{ - \sqrt{|g_{00}|g^3 g_\theta^2}\left[1 - \frac{g_{rr}}{|g_{00}|}\dot{r}^2 - \frac{g_{\rho\rho}}{|g_{00}|}\dot{\rho}^2 - \frac{g_{\phi\phi}}{|g_{00}|}\dot{\phi}^2\right]^{\frac{1}{2}}\left[1 + \frac{H_{345}^2}{g^3}\right]^{\frac{1}{2}}$$
$$+ R^3\rho^3 F_{345}\cos\varphi\dot{\phi} + \frac{1}{2}F_{345}f_{0*}\Big\}, \tag{25}$$

where $g_\theta = g_{\theta^1\theta^1} = f^{2/3}h^{-1/3}r^2\rho^2$.

IV. BRANE COSMOLOGY

Since we are interested in the cosmological evolution in terms of r, we consider the case when ρ = constant, i.e. $\dot{\rho} = 0$. This corresponds to the case when the probe universe is planar. In this case we can rewrite $\mathcal{L}$ as

$$\mathcal{L} = \Omega_2 T_{M5}\left\{-\sqrt{A(r) - B(r)\dot{r}^2 - D(r)\dot{\phi}^2} + G\dot{\phi} + \frac{1}{2}F_{345}f_{0*}\right\}, \tag{26}$$

where

$$A = |g_{00}|g_\theta^2(g^3 + H_{345}^2) = f r^4\rho^4\lambda_1^2,$$

$$
\begin{aligned}
B &= g_{rr} g_\theta^2 (g^3 + H_{345}^2) = f^2 r^4 \rho^4 \lambda_1^2, \\
D &= g_{\phi\phi} g_\theta^2 (g^3 + H_{345}^2) = f^2 r^6 \rho^4 (1-\rho^2) \lambda_1^2, \\
G &= R^3 \rho^3 F_{345} \cos\varphi. \qquad (27)
\end{aligned}
$$

The momenta and hamiltonian, divided by the overall factor $\Omega_2 T_{M5}$, are calculated as

$$
\begin{aligned}
p_r &= \frac{\partial \mathcal{L}}{\partial \dot{r}} = \frac{B(r)\dot{r}}{\sqrt{A(r) - B(r)\dot{r}^2 - D(r)\dot{\phi}^2}}, \\
p_\phi &= \frac{\partial \mathcal{L}}{\partial \dot{\phi}} = \frac{D(r)\dot{\phi}}{\sqrt{A(r) - B(r)\dot{r}^2 - D(r)\dot{\phi}^2}} + G, \qquad (28) \\
\mathcal{H} &= \dot{r} p_r + \dot{\rho} p_\rho + F_{0*} \frac{\partial \mathcal{L}}{\partial F_{0*}} = \frac{A(r)}{\sqrt{A(r) - B(r)\dot{r}^2 - D(r)\dot{\phi}^2}}.
\end{aligned}
$$

We require the conservation of energy as well as the angular momentum

$$
\mathcal{H} = \frac{A(r)}{\sqrt{A(r) - B(r)\dot{r}^2 - D(r)\dot{\phi}^2}} = E = \text{const}, \qquad (29)
$$

$$
p_\phi = \frac{D(r)\dot{\phi}}{\sqrt{A(r) - B(r)\dot{r}^2 - D(r)\dot{\phi}^2}} + G = \ell = \text{const}. \qquad (30)
$$

If we solve Eqs. (29) and (30) for $\dot{\phi}$ and $\dot{r}$, we have

$$
\dot{\phi}^2 = \left(\frac{A}{D}\right)^2 \left(\frac{\ell - G}{E}\right)^2, \qquad (31)
$$

$$
\dot{r}^2 = \frac{A}{B}\left\{1 - \frac{A}{E^2}\frac{D + (\ell - G)^2}{D}\right\}. \qquad (32)
$$

Since $\dot{r}^2 \geq 0$, we have the constraint for the allowed values of r

$$
\frac{A}{B}\left\{1 - \frac{A}{E^2}\frac{D + (\ell - G)^2}{D}\right\} \geq 0. \qquad (33)
$$

Using the expressions in Eq. (27), we can estimate the range of r where our formalism is valid

$$
\frac{r}{R} \lesssim \frac{E^2 R^2 (1-\rho^2)}{(\ell - R^3\rho^3 \cos\varphi F_{345})^2 + \rho^4 (1-\rho^2) R^6 \cos^2\varphi F_{345}^2} \equiv \frac{r_c}{R}. \qquad (34)
$$

The induced metric on the three-brane universe (= 5-brane/S^2) can be written as

$$ds_{4d}^2 = (-|g_{00}| + g_{rr}\dot{r}^2 + g_{\phi\phi}\dot{\phi}^2)dt^2 + g(r)[(dx^3)^2 + (dx^4)^2 + (dx^5)^2]. \tag{35}$$

Using Eqs. (31) and (32), this reduces to

$$ds_{4d}^2 = -|g_{00}|\frac{A}{E^2}dt^2 + g(r)[(dx^3)^2 + (dx^4)^2 + (dx^5)^2] \equiv -d\eta^2 + g(r(\eta))(d\vec{x})^2, \tag{36}$$

where we defined the cosmic time η as

$$d\eta = \frac{|g_{00}|^{1/2}A^{1/2}}{E}dt = \frac{|g_{00}|g^{3/2}g_\theta(1+\frac{H_{345}^2}{g^3})^{1/2}}{E}dt. \tag{37}$$

If we define the scale factor as $a^2 \equiv g$, we can calculate, from the analogue of the four-dimensional Friedman equation, the Hubble constant $H = \dot{a}/a$

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{1}{4|g_{00}|}\left(\frac{E^2}{B} - \frac{A}{B} - \frac{A}{D}\frac{(\ell - G)^2}{B}\right)\left(\frac{g'}{g}\right)^2, \tag{38}$$

where the dot denotes the derivative with respect to cosmic time η and the prime denotes the derivative with respect to r. The right hand side of Eq. (38) can be interpreted as the effective matter density on the probe 3-brane. Upon substituting the specific forms of B, C, D and G of Eq. (27) we have

$$\frac{8\pi}{3}\rho_{\rm eff} = \frac{1}{4|g_{00}|}\left(\frac{E^2}{\rho^4 r^4 f^2\lambda_1^2} - \frac{1}{f} - \frac{1}{(1-\rho^2)r^2 f}\frac{(\ell - G)^2}{\rho^4 r^4 f^2 \lambda_1^2}\right)\left(\frac{g'}{g}\right)^2. \tag{39}$$

Defining the dimensionless variable x as $x \equiv r/R$, we can write the effective matter density explicitly as

$$\begin{aligned}\frac{8\pi}{3}\rho_{\rm eff} &= \frac{1}{4R^2(\cos^2\varphi)^{1/3}}\frac{\{1+(1-\tan^2\varphi)x^3\}^2}{(1+x^3)^{7/3}(1+\sec^2\varphi x^3)^{7/3}} \\ &\times \left\{\frac{E^2\cos^2\varphi}{\rho^4 R^4 x^4}k(x,\varphi,F) - 1 - \frac{(\ell - R^3\rho^3 F_{345}\cos\varphi)^2\cos^2\varphi}{\rho^4(1-\rho^2)R^6x^3(1+x^3)}k(x,\varphi,F)\right\}, \end{aligned}\tag{40}$$

where

$$k(x,\varphi,F) = [1 - 2\sin\varphi\cos\varphi F_{345} + \cos^2\varphi(1+\cos^2\varphi x^{-3})F_{345}^2]^{-1}. \tag{41}$$

To obtain more transparent expression for the cosmological evolution, we express the effective matter density in terms of scale factor a as

$$\frac{8\pi}{3}\rho_{\rm eff} = \frac{1}{4R^2}\frac{a(f-1)^{8/3}}{f^{5/2}}(1-2a^3f^{1/3}\cos^2\varphi)^2$$
$$\times \Big[\frac{E^2}{\rho^4R^4}\frac{(f-1)^{4/3}}{a^3f^{1/2}}\{1+(\frac{F_{345}}{a^3}-\frac{\tan\varphi}{f^{1/2}})^2\}^{-1}-1$$
$$-\frac{(\ell-R^3\rho^3F_{345}\cos\varphi)^2}{(1-\rho^2)\rho^4R^6}\frac{(f-1)^2}{a^3f^{3/2}}\{1+(\frac{F_{345}}{a^3}-\frac{\tan\varphi}{f^{1/2}})^2\}^{-1}\Big], \tag{42}$$

where

$$f=\frac{1-2a^6\cos^2\varphi\sin^2\varphi+\sqrt{1-4a^6\cos^2\varphi\sin^2\varphi}}{2a^6\cos^4\varphi}. \tag{43}$$

Let us consider a limiting case to compare our expression with the result from type IIB background. We consider the case when there is no gauge field on the worldvolume, i.e., $F_{345}=0$. In this case the effective matter density is given by, taking the leading powers of the scale factor,

$$\frac{8\pi}{3}\rho_{\rm eff}\simeq\frac{1}{4R^2(\cos^2\varphi)^{1/3}}\Big[\frac{E^2}{\rho^4R^4(\cos^2\varphi)^{5/3}}\frac{1}{a^8}-1-\frac{\ell^2}{\rho^4(1-\rho^2)R^6\cos^2\varphi}\frac{1}{a^6}\Big]. \tag{44}$$

Near the horizon, the effective matter density is proportional to $\rho_{\rm eff}\sim a^{-8}$, which shows the same power behavior as the result from the type IIB background without any gauge field on the worldvolume [5]. Also the ℓ^2 term has the same sign and power behavior.

V. DISCUSSION

We searched the possibility of constructing the mirage cosmology in M-theory background. We considered the motion of a five-brane in the background formed by a stack of non-threshold (M2,M5) bound states. From the brane action in the PST formalism, we derived a Friedman-like equation. We took a limiting case and compared the result with the one from type IIB background. We conclude that the cosmological evolution by mirage energy density is a possible scenario in M-theory background.

As estimated in Eq. (34), our formalism on mirage cosmology holds for $r\lesssim r_c$. But this does not mean we can extend our result to the initial singularity where the effect of the back reaction is important. In mirage cosmology the initial singularity appears not because the solution is singular but because the effective field theory is not valid in this region. It is just an artifact of the low energy description [5]. The cosmological evolution from our result can be summarized as follows. When the probe brane is near the (M2,M5) bound states, the probe brane expands mainly due to mirage energy density. In this region the universe expands very rapidly ($\rho_{\rm eff} \sim a^{-8}$). As the brane universe moves away from the background bound states ($r \gtrsim r_c$), the effect of the background branes to the probe brane will not be strong enough to drive the inflation. Then our formalism on mirage cosmology is not valid any more. In this region the matter density of the probe universe itself will drive the cosmological expansion and the rate of expansion will be slower than the one by mirage energy density.

Although it is an important open problem how to study the back reaction of the probe brane, we did not consider the back reaction of the probe brane to the background geometry. When ℓ^2 term dominates the effective density is negative and we have contraction rather than inflation. We hope this fact might be improved if we consider the back reaction. In our presentation, we considered the motion of probe brane with constant ρ ($\dot{\rho} = 0$). It would be an interesting topic if one studies the mirage cosmology with constant r ($\dot{r} = 0$). In this case, we expect that one could construct the mirage cosmology of closed universe similar to the case in Ref. [11]. It is known that a Friedman-type evolution in brane cosmology is equivalent to the formalism of varying speed of light [6]. One can also study this model in this context.

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